

Fundamentals of Algebraic Modeling

An Introduction to Mathematical Modeling
with Algebra and Statistics

Fifth Edition

DANIEL L. TIMMONS

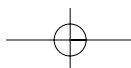
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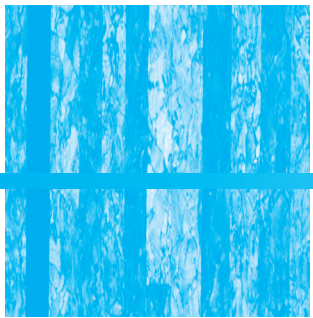
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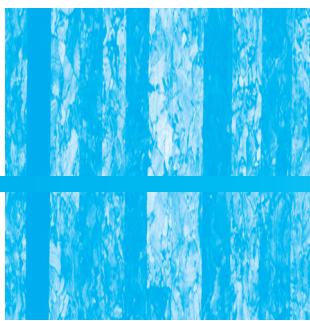
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Preface

“The longer mathematics lives the more abstract—and therefore, possibly also the more practical—it becomes.” – Eric Temple Bell (1883–1960)

TO THE INSTRUCTOR

There is no doubt that mathematics has become increasingly more important in our ever-changing world. For most people, the usefulness of mathematics lies in its applications to practical situations. Our goal in writing this book is to get students to think of mathematics as a useful tool in their chosen occupations and in their everyday lives.

This book was written and designed for students in a two-year associate in arts curriculum who are not planning additional course work in mathematics. We have written the book in “nonthreatening” mathematical language so that students who have been previously fearful of or intimidated by mathematics will be able to comprehend the concepts presented in the text. We have tried to write it in such a manner that students with backgrounds in fundamental algebra can understand and learn the ideas we have presented.

Various types of problems are included throughout the book in order to attempt to make students aware of their own thought processes. Our intent is to teach students how to approach a variety of problems with some basic skills and a plan for success. We hope that students will learn to use, or develop and then test, mathematical models against reality. Further, we have tried to be sensitive to various student learning styles. This was done by including problems in many formats (graphical, numerical, and symbolic) in order to give students many different opportunities to “see” the mathematics.

IN THE FIFTH EDITION

The elements that proved successful in previous editions remain in this edition. However, we have reordered several sections and added some new topics. Many of the problem sets have had extensive revision with expanded problem sets including many new problems. The answers to the odd exercises are included in the answer key in the back of the book, with answers to all problems in the Chapter Reviews and Chapter Tests in the key. Additional lab activities have been included in many of the chapters. Here is a list of the major changes included in the fifth edition.

- An extensive revision of problem sets has been done in Chapter 1 to include problems that are applicable in many areas of real life. The topic of scientific notation has also been added to this chapter.
- Chapter 2 includes a new section demonstrating some applications and uses of graphs.
- The section in Chapter 3 on nonlinear functions from the fourth edition has been expanded and divided into two sections in this chapter. One focuses on quadratic functions and the other focuses on other nonlinear functions such as exponential functions and power function(s).
- Chapter 4 on consumer finance focuses on models in the business world. New topics include insurance options and rate, purchasing versus leasing a car, and stocks, mutual funds, and bonds.
- Chapter 5 is a completely new chapter that was added to illustrate some additional topics related to modeling. The geometry sections are in this chapter as well as sections on modeling and patterns in architecture and in music. The sections on architecture include perspective, symmetry, scale, and proportion.
- We have retained the explanation and use of Cramer's Rule in Chapter 6 simply because it introduces a simple matrix to students and is an alternative way to solve systems that many students have not been exposed to in other courses.
- Chapter 7 on probability models includes an introduction to sets in the first section. The inclusion of Venn diagrams with the topics of union and intersection is designed to help students understand more clearly the concepts of "or" and "and" problems in probability. The topic of odds has been included in the section on theoretical probability instead of being presented as a separate section.
- Data sets and problems in Chapter 8, Modeling with Statistics, have been updated and expanded.

FEATURES OF THE BOOK

Laboratory Exercises

The American Mathematical Association of Two-Year Colleges (AMATYC) in its publication *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus* recommends that mathematics be taught as a laboratory discipline where students are involved in guided hands-on activities. We have included laboratory exercises at the end of each chapter and a wide variety of other activities in the ancillary materials available with this text. Some are designed to be completed as individual assignments and others require group work. There are assignments that require access to a computer lab and several that require online work. We have tried to make the labs versatile so the instructor can use any technology available. However, instructors are not required to have technology in order to teach a successful course using this book.

Calculator Mini-Lessons

AMATYC also recommends in *Crossroads* the routine use of calculators in the classroom. We have created calculator mini-lessons throughout the book to aid students who are unfamiliar with calculators in becoming more proficient. Instructions for both a standard scientific calculator and a graphing calculator are provided. The use

of graphing calculators is recommended in several areas such as graphing nonlinear functions, science and technology applications, and linear regression. However, the use of a graphing calculator is not essential to the successful completion of a course using this book.

Chapter Summaries

At the end of each chapter is a chapter summary listing key terms and formulas and points to remember.

Examples

We have included clear step-by-step examples in each section to illustrate the concepts and skills being introduced. More examples have been added to the material in several sections based on suggestions by other instructors.

Answer Key

The answer key provides odd-numbered answers to the practice sets. The focus of this book is understanding the modeling process involved in solving a problem. Having answers available helps students know if their thinking process has been correct. All answers are provided for the chapter reviews and chapter tests.

FOR THE INSTRUCTOR

Instructor's Resource Manual

This supplement includes additional labs and group activities for each chapter of the text, lab notes for the instructor, a test bank with five tests per chapter as well as three final exams, and solutions to all the problems in the text. The manual also includes worksheets that can be given to students for additional practice. There is also a graphing calculator quick reference guide.

ExamView Computerized Testing

Create, deliver, and customize tests (both print and online) in minutes with this easy-to-use assessment and tutorial system. Includes algorithmically generated questions.

Instructor's Companion Web Site

The Instructor's Companion Site includes tools for class preparation, including a lesson planning guide and tutorial quizzes.

FOR THE STUDENTS

Student Companion Web Site

The Student Companion Site includes self-assessment quizzes, tutorial quizzes to practice skills, and a math glossary in English and Spanish.

Student Solutions Manual

The Student Solutions Manual provides worked-out solutions to the odd-numbered problems in the text.

TO THE STUDENT

Why take math courses? You may have asked yourself or your advisor this very question. Perhaps you asked because you don't see a need for math in your primary area of study. Perhaps you asked because you have always feared mathematics. Well, no matter what your math history has been, your math future can be better. As you start this new course, try to cultivate a positive attitude and look at the tips we offer for success.

Math can be thought of as a tool. It does have a practical value in your daily life as well as in most professions. In some fields such as engineering, accounting, business, drafting, welding, carpentry, and nursing, the connection to mathematics is obvious. In others such as music, art, history, criminal justice, and early childhood education, the connection is not as clear. But, we assure you, there is one. The logic developed by solving mathematical problems can be useful in all professions. For example, those in the criminal justice field must put together facts in a logical way and come to a solution for the crimes they investigate. This involves mathematical processes.

Overcoming anxiety about math is not easy for most students. However, developing a positive attitude, improving your study habits, and making a commitment to yourself to succeed can all help. Enroll in any study-skills courses offered and take advantage of any tutoring services provided by your college. Do this on the first day of class, not after you've done poorly on two or three tests. To reduce your math anxiety, try these tips.

- Be well prepared for tests. Practice “taking tests” at home with a timer.
- Write down memory cues before beginning a test.
- Begin a test by first doing the problems with which you have the least trouble.
- Take advantage of all available help (tutoring services, skills lab, instructor office hours).
- Learn from your mistakes by reworking all problems missed on a test or homework assignment.
- Take math courses in the fall or spring semester—not during a short summer term.
- If you must take several math courses, take them in consecutive semesters.
- Form study groups to study outside of class time.

You *can* choose to be successful by using these tips and giving this course the time necessary to master the material.

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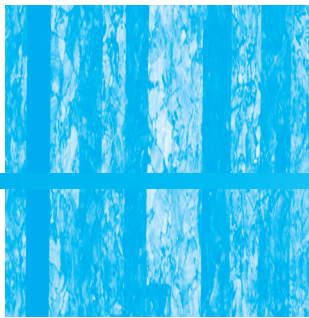
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Dan Timmons
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 2009



Keys to Success

BEFORE CLASS STARTS

- ✓ Find a quiet, comfortable place to work outside of class.
- ✓ Make both short-term and long-term study schedules.
- ✓ Discourage interruptions.
- ✓ Take short breaks occasionally.
- ✓ Do not procrastinate.
- ✓ Give yourself some warm-up time by working simple problems.

IN CLASS

- ✓ Attend class regularly.
- ✓ Ask questions early in the term.
- ✓ Listen for critical points.
- ✓ Lost? Mark your location in your notes and see your instructor during the next available office hour or at the end of class, if time permits.
- ✓ Review your notes from the previous class before going to class again.
- ✓ Be sure to read the text sections that correspond to your lecture notes.
- ✓ Form a study group with some of your classmates.

ABOUT THOSE CLASS NOTES

- ✓ Your notes are your links between your class and your textbook.
- ✓ Never write at the expense of listening.
- ✓ Forget about correct grammar while taking notes.
- ✓ Use a lot of abbreviations.
- ✓ Be sure to copy down class examples.
- ✓ You may even want to rewrite your class notes to make them clearer and neater for future reference.
- ✓ Compare your notes with those of some of your classmates; they may have gotten some points that you missed and vice versa.

PROPERLY USE THIS TEXTBOOK

- ✓ Read the section due for lecture before class.
- ✓ Read each section twice, first quickly and then slowly while referring to your class notes.

xii **Keys to Success**

- ✓ Writing down a concept or idea is definitely linked to your thinking processes, so write things down as they occur to you in your reading.
- ✓ Write notes to yourself in your textbook margins.
- ✓ Look up the definition of unfamiliar terms.
- ✓ Highlight sparingly.

LOVE THOSE WORD PROBLEMS

- ✓ Think about the problem before jumping into a solution.
- ✓ Be sure to clearly delineate the questions to be answered.
- ✓ Break long, complicated problems into parts.
- ✓ Work with your study group but remember that you must be able to solve problems on your own at test time.
- ✓ Ask for help (from your instructor, tutor, classmates) when you need it.

TEST PREPARATION

- ✓ Be sure you know what topics are to be covered.
- ✓ You and your study group members should make up reasonable questions for each other to practice.
- ✓ Review old quizzes or tests if you can.
- ✓ Reread those marginal notes you made in your textbook, particularly those that indicate weakness.
- ✓ Honestly admit your weaknesses and work on strengthening them.